

COMPUTATION OF THE VELOCITY PROFILE FOR NONLINEARLY VISCOUS FLUID
FLOW IN SPIRAL CHANNELS WITH LOW REYNOLDS NUMBERS

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Two methods are presented for solving the problem of nonlinear viscous fluid flow in spiral channels of arbitrary transverse section in the Stokes approximation, variational and iterational.

Formulation of the Problem

Let us consider the problem of finding the velocity profile being shaped during stationary flow of a nonlinear viscous fluid in a spiral channel of arbitrary transverse section in a noninertial (Stokes) approximation.

The urgency of the problem is based on the fact that the most effective method of intensifying the convective heat and mass transfer for media with high viscosity and quite definite viscosity anomalies is the application of spiral ribbing of the channels [1].

Earlier the existence and uniqueness of the solution of the formulated problem was proved in [2]. It was simultaneously shown in [2] that independently of the shape of its transverse section a spiral channel possesses a one-parameter symmetry group, shifts along spiral lines.

In this connection, we introduce a spiral coordinate system related to the cylindrical system by the relationships

$$q^1 = r, \quad q^2 = \varphi - \frac{2\pi}{S} z, \quad q^3 = z, \quad (1)$$

$$V_1 = V_r, \quad V_2 = rV_\varphi, \quad V_3 = V_z + \frac{2\pi}{S} rV_\varphi.$$

In this case fixing q^1 and q^2 yields a spiral line while the operator $\partial/\partial q^3$ is the derivative in the direction of the spiral lines, i.e., the third spiral coordinate. The velocity components V_1, V_2, V_3 in the spiral coordinate system introduced in this manner will be self-similar in the variable q^3 .

Then the system of motion and continuity equations describing this problem can be represented by using vortex-stream function variables in the form

$$\frac{1}{q^1} \frac{\partial}{\partial q^1} \left(\mu(I_2) q^1 \frac{\partial \omega_3}{\partial q^1} \right) + \frac{A}{(q^1)^2} \frac{\partial}{\partial q^2} \left(\mu(I_2) \frac{\partial \omega_3}{\partial q^2} \right) + \frac{4\pi}{Sq^1} \mu(I_2) \left[\frac{\partial}{\partial q^1} \left(q^1 \frac{\partial}{\partial q^1} \left(\frac{V_3}{A} \right) \right) + \right.$$

$$\left. + \frac{A}{(q^1)^2} \frac{\partial}{\partial q^2} \left(q^1 \frac{\partial}{\partial q^2} \left(\frac{V_3}{A} \right) \right) + \frac{2\pi}{S} \frac{\partial}{\partial q^1} \left(q^1 \frac{4\pi}{SA^2} V_3 - \frac{\omega_3 (q^1)^2}{A} \right) \right] + (\text{rot } B)_3 = 0, \quad (2)$$

$$\omega_3 = \frac{4\pi}{SA} V_3 - \frac{A}{q^1} \left[\frac{\partial}{\partial q^1} \left(\frac{q^1}{A} \frac{\partial \psi}{\partial q^1} \right) + \frac{A}{(q^1)^2} \frac{\partial}{\partial q^2} \left(\frac{q^1}{A} \frac{\partial \psi}{\partial q^2} \right) \right], \quad (3)$$

$$\frac{\partial P}{\partial q^3} = \frac{1}{q^1} \frac{\partial}{\partial q^1} \left(\mu(I_2) q^1 A \frac{\partial}{\partial q^1} \left(\frac{V_3}{A} \right) \right) + \frac{A}{(q^1)^2} \frac{\partial}{\partial q^2} \left(\mu(I_2) A \times \right.$$

$$\left. \times \frac{\partial}{\partial q^2} \left(\frac{V_3}{A} \right) \right) + \frac{4\pi}{Sq^1} \left[\frac{\partial}{\partial q^1} \left(\mu(I_2) \frac{q^1}{A} \frac{\partial \psi}{\partial q^1} \right) + \frac{A}{(q^1)^2} \frac{\partial}{\partial q^2} \left(\mu(I_2) \frac{q^1}{A} \frac{\partial \psi}{\partial q^2} \right) \right] \quad (4)$$

with the "adhesion" boundary condition on the channel wall and the second invariant of the strain rate tensor equal to

$$\begin{aligned}
 I_2 = 2 \operatorname{tr} (D^2) = & 2 \left(\frac{\partial}{\partial q^1} \left(\frac{1}{q^1} \frac{\partial \psi}{\partial q^2} \right) \right)^2 + \frac{A}{(q^1)^2} \left(\frac{\partial}{\partial q^2} \left(\frac{1}{q^1} \frac{\partial \psi}{\partial q^2} \right) \right)^2 + \\
 & + \frac{2}{(q^1)^2} \left(\frac{1}{q^1} \frac{\partial \psi}{\partial q^2} \right)^2 - 2 \frac{\partial}{\partial q^2} \left(\frac{1}{q^1} \frac{\partial \psi}{\partial q^2} \right) \frac{\partial}{\partial q^1} \left(\frac{1}{q^1} \frac{\partial \psi}{\partial q^1} \right) - \\
 & - \frac{4}{A (q^1)^2} \frac{\partial \psi}{\partial q^2} \frac{\partial}{\partial q^2} \left(\frac{1}{q^1} \frac{\partial \psi}{\partial q^1} \right) + \frac{4K}{(q^1)^2} \frac{\partial \psi}{\partial q^2} \frac{\partial}{\partial q^2} \left(\frac{V_3}{A} \right) + \\
 & + A (q^1)^2 \left(\frac{\partial}{\partial q^1} \left(\frac{1}{A q^1} \frac{\partial \psi}{\partial q^1} \right) \right)^2 + 2 \left(\frac{\partial}{\partial q^2} \left(\frac{1}{q^1} \frac{\partial \psi}{\partial q^1} \right) \right)^2 + \\
 & + \frac{4K}{A} \left(\frac{\partial \psi}{\partial q^1} \right)^2 + \frac{4K^2 (q^1)^2}{A} \frac{\partial \psi}{\partial q^1} \frac{\partial}{\partial q^1} \left(\frac{1}{q^1 A} \frac{\partial \psi}{\partial q^1} \right) + \\
 & + \frac{4K}{A} \frac{\partial \psi}{\partial q^1} \frac{\partial}{\partial q^1} \left(\frac{V_3}{A} \right) + A \left(\frac{\partial}{\partial q^1} \left(\frac{V_3}{A} \right) \right)^2 + \frac{A^2}{(q^1)^2} \left(\frac{\partial}{\partial q^2} \left(\frac{V_3}{A} \right) \right)^2.
 \end{aligned} \tag{5}$$

VARIATIONAL METHOD

The functional of the action for the formulated problem is defined in [2] as

$$F(\bar{V}) = \frac{1}{2} \iiint_{\bar{K}} \mu^* (I_2) dv - 2 \frac{\partial P}{\partial q^3} \iiint_{\bar{K}} (\bar{V}, \bar{n}) dv, \tag{6}$$

and then is reduced to the form

$$F(\bar{V}) = \frac{S}{2} \iint_{\Omega} \mu^* (I_2) d\Omega - 2 \frac{\partial P}{\partial q^3} S \int_{\Omega} V_z d\Omega, \tag{7}$$

when the self-similarity of the velocity vector relative to the spiral coordinate is taken into account, where it is proved in [3] that it is weakly continuous and has a lower bound, and consequently, a unique stationary point in H .

To assure the stability of the variational solution, we represent the function realizing the extremum of the functional (7) (for each of the velocity components) in the form

$$V_j = \sum_1^m A_n^j f_n. \tag{8}$$

We use the eigenfunctions of contiguous operators for the problem (2)-(4) under consideration in the domain Ω as the complete and linearly independent system of coordinate functions f_n .

Let us examine two spiral channel shapes, a tube with a tape spiral insert and a coaxial channel with spiral ribbing of the annular gap. For the first construction (neglecting half the tape thickness, a semicircle in the transverse section) the eigenfunctions of the contiguous (Poisson) operator have the form

$$f_n = C_{\text{cr}} J_k \left(v_{\text{cr}} \frac{q^1}{R} \right) \sin(kq^2),$$

where C_{cr} is selected from the condition

$$\|f_n\|^2 = 1 = \frac{\pi}{2} C_{\text{cr}}^2 \int_0^R \frac{q^1}{R^2} \left(v_{\text{cr}}^2 J_k^2 \left(v_{\text{cr}} \frac{q^1}{R} \right) + \frac{R^2 k^2}{(q^1)^2} J_k^2 \left(v_{\text{cr}} \frac{q^1}{R} \right) \right) dq^1.$$

For the second construction (a rectangle in transverse section)

$$f_n = \frac{2}{\pi} \left(\frac{k^2}{(R_2 - R_1)^2} + \frac{p^2 (R_2 + R_1)^2}{4(\varphi_2 - \varphi_1)^2} \right)^{-\frac{1}{2}} \sin \frac{k\pi(q^1 - R_1)}{(R_2 - R_1)} \sin \frac{p\pi(q^2 - \varphi_1)}{(\varphi_2 - \varphi_1)}.$$

The coefficients of the expansion in (8) are found from the condition of minimum of the functional (7) by using the Ritz method

$$\frac{\partial F(\bar{V})}{\partial A_n^j} = \frac{S}{2} \iint_{\Omega} \mu(I_2) \frac{\partial I_2}{\partial A_n^j} d\Omega - \frac{\partial P}{\partial q^3} S \iint_{\Omega} \frac{\partial V_z}{\partial A_n^j} d\Omega \quad (n=1, 2, \dots, m_j). \quad (9)$$

After evaluation of the intergrands and appropriate manipulations the system (9) is written in the form of systems of nonlinear equations in the coefficients of the expansion

$$\sum_1^{m_j} \left(A_n^j \hat{\theta}_{nm} + \frac{\partial P}{\partial q^3} E_n^j \right) = 0, \quad j = 1, 2, 3, \quad (10)$$

where

$$\hat{\theta}_{nm} = \iint_{\Omega} \mu(I_2) \xi_{nm}(q^1, q^2) d\Omega; \quad E_n^j = 2S \iint_{\Omega} \frac{\partial V_z}{\partial A_n^j} d\Omega;$$

$\xi_{nm}(q^1, q^2)$ are the results of calculating the expressions in the square brackets in (9).

A Kachanov calculational process [4] is used for the numerical realization of the variational method, whereupon the system (10) is linearized in the first stage of the calculation by conferring a certain value $\mu = \text{const}$ on $\mu(I_2)$. In a first approximation A_n^j and V_j are determined here. After evaluating (5) and the effective viscosity by means of a selected or given rheological equation, $\hat{\theta}_{nm}$, A_n^j , and V_j are again computed. Therefore, the system (10) is linearized in each stage of the calculation, which is equivalent to freezing the coefficients $\mu(I_2)$ in (2)-(4). Solution of the linear system (10) in each stage of the calculations is by the Gauss method, where repeated Gauss quadratures are used here to evaluate $\hat{\theta}_{nm}$. Evidently, the Newtonian viscosity, usually μ_0 , is given as the first approximation of $\mu(I_2)$ independently of the rheological model. In the first stage of the calculations the profiles of the velocity components correspond to the Newtonian. If the sequence of solutions of the systems (10) tend to a certain limit in $\mu(I_2)$ while the systems themselves tend to a limit in A_n^j in each stage of the calculations, then this is indeed the solution of the problem.

ITERATION METHOD

To construct an algorithm of pure iteration type, we represent all three equations of the system (2)-(4) that have approximately identical structure in the following form according to [5]

$$\frac{\partial}{\partial q^1} \left(\sigma_{1i} \frac{\partial W_i}{\partial q^1} \right) + \frac{\partial}{\partial q^2} \left(\sigma_{2i} \frac{\partial W_i}{\partial q^2} \right) = -f(W_j, W_k), \quad j, k \neq i, \quad (11)$$

where σ_{ji} are functions of I_2 and the coordinates in the general case [just functions of the coordinates q^1 and q^2 for equation (3)], and W_i is understood to be ω_3 for (2), ψ for (3), and V_3 for (4).

Each of the equations of the system (2)-(4) reduced to the form (11) is solved individually, where the W_j , W_k ($j, k \neq i$) from the preceding stage of the calculations are used here to find the values. As any other iteration approach to the solution of such nonlinear problems, the algorithm proposed for the computation consists of an infinite number of steps of approximation to the desired solution.

An important step in the construction of iteration structures of solutions, especially of nonlinear problems, is the selection or finding of the first approximation. The substitution

$$\sigma_{ji} \frac{\partial W_i}{\partial q^j} = \frac{\partial U}{\partial q^j}, \quad j = 1, 2, \quad (12)$$

is highly recommended in [6] for finding the first approximation when solving equations of the form (11) and for the numerical realization of the problem of nonlinear viscous fluid flow in a cylindrical channel of arbitrary transverse section and would simultaneously permit an explicit solution to be obtained in the Newtonian case.

In this situation, i.e., for a flow in a spiral channel of arbitrary transverse section, we will use the substitution (12) to find V_3 in a first approximation by means of (4) by considering $\psi = 0$ and $\omega_3 = -(4\pi/AS)V_3$.

Then the matrices of values of I_2 are evaluated according to (5) and of $\mu(I_2)$ by using a rheological model of a fluid. Substitution of the values of $\mu(I_2)$ into the equations of the system (2)-(4) reduced to the form (11) permits freezing the coefficients of (11) and solving them successively by known methods.

We use the iteration method of variable directions (MVD) which has the following form for the problem under consideration

$$\begin{aligned} \frac{Y^{k+1/2} - Y^k}{\tau^1} &= \Lambda_1 Y^{k+1/2} + \Lambda_2 Y^k + f^k/2, \\ \frac{Y^{k+1} - Y^{k+1/2}}{\tau^2} &= \Lambda_1 Y^{k+1/2} + \Lambda_2 Y^{k+1} + f^k/2, \end{aligned} \quad (13)$$

where

$$\begin{aligned} \Lambda_1 Y_{ij} &= \frac{1}{h_1^2} (Y_{ij-1} k_{1ij} - Y_{ij} (k_{1ij} + k_{1ij+1}) + Y_{ij+1} k_{1ij+1}); \\ k_{1ij} &= 0,5 (\sigma_1 (q_{i-1}^1, q_j^2) + \sigma_1 (q_i^1, q_j^2)); \\ \Lambda_2 Y_{ij} &= \frac{1}{h_2^2} (Y_{i-1j} k_{2ij} - Y_{ij} (k_{2ij} + k_{2i+1j}) + Y_{i+1j} k_{2i+1j}); \\ k_{2ij} &= 0,5 (\sigma_2 (q_i^1, q_{j-1}^2) + \sigma_2 (q_i^1, q_j^2)), \end{aligned}$$

for the numerical realization. Each of the equations of the system (13) is solved by a standard factorization method, where the factorization coefficients for the first equation acquire the form

$$\begin{aligned} A_i &= \frac{k_{2ij}}{h_2^2}, \quad B_i = \frac{k_{2i+1j}}{h_2^2}, \quad C_i = A_i + B_i + \frac{1}{\tau^2}, \\ F_i &= f_{ij}^k/2 + \frac{Y_{ij}^{k+1/2}}{\tau^2} + \frac{1}{h_1^2} [Y_{ij-1}^{k+1/2} k_{1ij} - Y_{ij}^{k+1/2} (k_{1ij} + k_{1ij+1}) + Y_{ij+1}^{k+1/2} k_{1ij+1}], \end{aligned}$$

and for the second equation

$$\begin{aligned} A_j &= \frac{k_{1ij}}{h_1^2}, \quad B_j = \frac{k_{1ij+1}}{h_1^2}, \quad C_j = A_j + B_j + \frac{1}{\tau^1}, \\ F_j &= f_{ij}^k/2 + \frac{Y_{ij}^k}{\tau^1} + \frac{1}{h_2^2} [Y_{i-1j}^k k_{2ij} - Y_{ij}^k (k_{2ij} + k_{2i+1j}) + Y_{i+1j}^k k_{2i+1j}]. \end{aligned}$$

When formulating problems in the variables vortex-stream function, the question of the boundary conditions for these variables remains. We obtain from the adhesion condition

$$V_s|_{\Gamma} = 0, \quad \frac{\partial \psi}{\partial \tau}|_{\Gamma} = 0, \quad \frac{\partial \psi}{\partial n}|_{\Gamma} = 0, \quad (14)$$

where τ and n are the tangent and normal to the channel contour.

Using the condition (14) and equation (2) of the initial system, by known reasoning [7] we obtain the conditions:

for (3)

$$V_s|_R = 0, \quad \psi|_R = 0, \quad (15)$$

for (4)

$$\omega_s|_R = -\frac{A}{q^1} \left[\frac{\partial}{\partial q^1} \left(\frac{q^1}{A} \frac{\partial \psi}{\partial q^1} \right) + \frac{A}{(q^1)^2} \frac{\partial}{\partial q^2} \left(\frac{q^1}{A} \frac{\partial \psi}{\partial q^2} \right) \right] \Big|_R, \quad (16)$$

$$\frac{\partial \varphi}{\partial n} \Big|_R = 0.$$

Therefore, the set of boundary conditions (14)-(16) permits complete formulation of this problem in the vortex-stream function variables.

The iteration parameters (fictitious time) τ^1 and τ^2 are selected according to [8] in such a manner that the quantity of internal iterations is minimal.

After the external iterations $W_i^n(q^1, q^2)$ are calculated in the second stage ($n = 2$) for all i the matrices I_2 and $\mu(I_2)$ are again evaluated. The process is continued until a given degree of relative error is achieved.

COMPARATIVE ANALYSIS OF THE METHODS AND RESULTS OF THE CALCULATIONS

As has already been mentioned, a tube with a spiral tape insert and a coaxial channel with spiral ribbing of the gap were examined as specific spiral channel shapes. The functions used for the substitution (12) are presented in [6] for both shapes.

The generalized Kutateladze-Khabakhpashevaya rheological equation [9] was used as the specific dependence $\mu(I_2)$

$$d\varphi_* = -\varphi_*^n d\tau_*, \quad (17)$$

in the exponential form (for large n)

$$\varphi_* = \exp(-\tau_*),$$

$$\varphi_* = (\varphi_\infty - \Phi)/(\varphi_\infty - \varphi_0), \quad \tau_* = \theta(\tau - \tau_0)/(\varphi_\infty - \varphi_0), \quad \Phi = \frac{1}{\mu(I_2)}. \quad (18)$$

The case of model fluid flow subject to (18) with the parameters $\theta = 0.1981 (\text{Pa}^2 \cdot \text{sec})^{-1}$, $\varphi_0 = 1.9 \text{ l/Pa} \cdot \text{sec}$, $\varphi_\infty = 1/\text{Pa} \cdot \text{sec}$, $\tau_0 = 0$ for $\partial P/\partial q^3 = 60 \text{ m/m}^3$ was considered for both methods. The channel dimensions were: tube with tape insert $R = 0.006 \text{ m}$, $S = 0.08 \text{ m}$; coaxial spiral channel $R_1 = 0.021 \text{ m}$, $R_2 = 0.036 \text{ m}$, $S = 0.08 \text{ m}$.

Results of computations of the velocity fields in the tube with the tape spiral insert and in the coaxial spiral channel are presented in the figure. The velocity profiles obtained are identical for both methods.

A comparative analysis of the two methods of computation showed that the purely iteration approach possesses a very much greater rate of convergence and smaller time expenditures per computation.

Solution of the problem in the variational formulation for the flow conditions presented and the spiral channel dimensions required six iterations to reach a relative error of $\epsilon = 10^{-4}$ in the effective viscosity for both channel shapes. The awkwardness of the computations of the system (10) and the extremely poor convergence of calculations because of the presence of the integral sines should be noted. On the other hand, the traditional a priori representation of the basis functions (8) in polynomial form considerably shortens the calculation process but is distinguished by poor stability, especially for domains possessing incomplete symmetry. Thus, for instance, for the considered flow examples and to assure a relative error of $\epsilon = 10^{-4}$ in the expansion coefficients A_n^j in (8), a minimal number of equations in the system (10) of 27 in each flow velocity component would be required for the tube with the tape insert and 24 for the coaxial spiral channel. An increase in the computation accuracy to $\epsilon = 10^{-6}$ would respectively result in 42 and 36 equations in each velocity component.

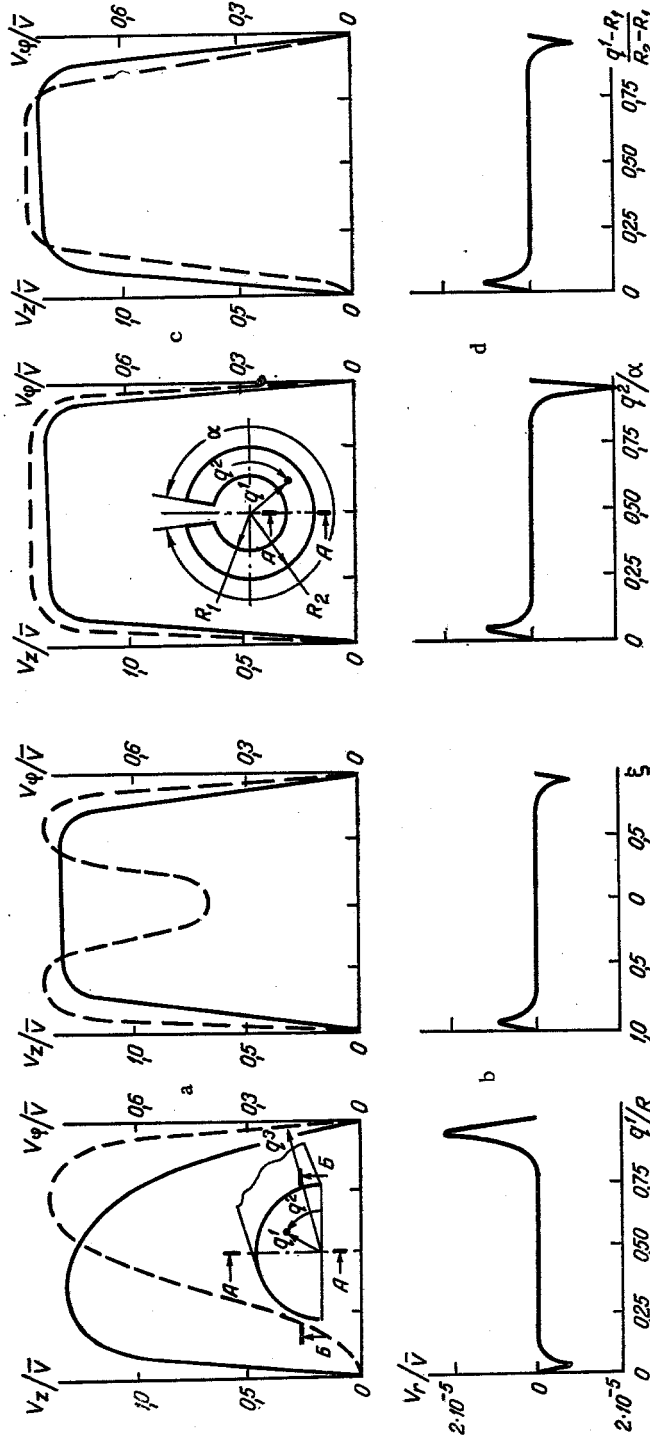


Fig. 1. Computed dimensionless diagrams of the axial (solid lines a, c), circumferential (dashes a, c) and radial (b, d) velocity components in the sections A-A and B-B ($r = R/3$) of the tube with tape insert (a, b) and in sections $r = (R_1 + R_2)/2$ and A-A' of the coaxial channel with spiral ribbing (c, d).

Realization of the problem in a purely iterational formula for completely identical flow conditions would require (for the same computation accuracy $\varepsilon = 10^{-4}$) four external iterations in effective viscosity for both channel shapes and nine and five internal iterations (by the MVD method) for the tube with the tape insert and the coaxial spiral channel. An increase in the computation accuracy to $\varepsilon = 10^{-6}$ would also result in just growth of the number of internal standard iterations to 12 and 6, respectively.

Estimation of the influence of the rheological model parameters and of the hydrodynamic flow characteristics on the indices of the calculational iteration process was confirmed by deductions elucidated in [6].

NOTATION

q^1, q^2, q^3 , running spiral coordinates; r, φ, z , running cylindrical coordinates; S , step of the spiral pitch; V_r, V_φ, V_z , radial, circumferential, and axial velocity components; V_1, V_2, V_3 , velocity components in the spiral coordinate system; \bar{V} , velocity vector; μ , effective viscosity; $I_2 = 2 \text{tr}(D^2)$, second invariant of the strain rate tensor; ψ , stream function introduced by the relationships $V^1 = (1/q^1)(\partial\psi/\partial q^2)$, $V^2 = -(1/q^1)(\partial\psi/\partial q^1)$; ρ , density; ω_3 , third component in the spiral coordinate system; $A = 1 + (2\pi/S)^2(q^1)^2$; D , strain rate tensor; Ω , domain with boundary Ω ; $F(\bar{V})$, the functional to be minimized; K , volume of the spiral channel included between sections orthogonal to the channel axis and at a distance S from each other; A_n^j , coefficients of the velocity component expansion in the system of basis functions f_n ; H , Hilbert space of solenoidal vector-functions with spiral symmetry condition; f_n , a complete and linearly independent system of coordinate functions used as basis functions; J_k , Bessel functions; v_{cr} , root of the Bessel functions; R , tube radius; R_1, R_2 , internal and external radii of the coaxial channel; φ_1, φ_2 , values of the polar angle corresponding to the thickness of the spiral ribbing in the coaxial channel; μ_0 , greatest Newtonian viscosity (as $\tau \rightarrow 0$); τ , intensity of the tangential shear stress; $B = \text{grad}(\mu, D)$; ϕ , fluid yield; $\varphi_0, \varphi_\infty$, yield as $\tau \rightarrow 0$ and $\tau \rightarrow \infty$; θ, τ_0 , measure and limit of the structural stability of the fluid; μ^* , primitive of $\mu(I_2)$ such that $\mu^*(0) = 0$; n, m , numeral and number of series of desired function expansions; U , an auxiliary function that is a solution of the Dirichlet problem for the Poisson equation; Y , difference analog of the desired function; Λ , difference analog of the differential equation operators; τ^1, τ^2 , iteration parameters (fictitious time); h_1, h_2 , steps of the difference mesh in the q^1 and q^2 directions; A, B, C, F , coefficients of the factorization method; σ_{ji} , variable coefficients of an elliptical equation that is functions of I_2 and the coordinates; ε , relative error, $K = 2\pi/S$. Subscripts: j, i , numbers of the mesh matrix nodes; n , number of the external iteration, and k , number of the iteration of the method of variable directions.

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